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Notched tensile strength of various fibre reinforced metal laminates

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Abstract—Modifications in Whitney–Nuismer (WN) fracture models known as the point stress criterion and the average stress criterion are proposed for notched strength estimations of laminates. Applicability of the proposed procedure is examined by considering the residual strength data on carbon fibre reinforced metal laminates, aramid reinforced aluminium laminates and glass fibre reinforced aluminium laminates having saw-cuts and circular holes of various sizes. The analytical results obtained in the present study are close to the test results. This study confirms that the present modification in WN fracture models for fibre reinforced composites can be used for notched strength evaluation of FRMLs.

Keywords: Fibre reinforced metal laminates; notched tensile strength; circular hole; saw-cut; point stress criterion; average stress criterion; stress intensity factor.

1. INTRODUCTION

Fibre-reinforced metal laminates (FRMLs) which are expected to possess high resistance to fatigue crack propagation in addition to high fracture toughness, are required in fatigue-critical structures such as the fuselage, lower wing skin and tail skins of aircraft. The requirement has lead to the development of CFRML (Carbon Reinforced Aluminium Laminate), ARALL (aramid fibres instead of carbon fibres) and GLARE (glass fibres instead of carbon fibres). Lawcock *et al*. [1] have investigated the residual strength of CFRML, ARALL and GLARE specimens with saw-cuts and circular holes of various sizes. They applied the average stress criterion (ASC) developed by Whitney and Nuismer [2, 3] for fibre reinforced composites to correlate residual strength data. The ASC is a two parameter model, based on the stress distribution adjacent to the discontinuity. It is assumed that fracture occurs when the average stress ahead of the discontinuty reaches the unnotched strength, σ_0, over the characteristic distance, a_0. Only one experimental notched test result is required to determine the characteristic distance, however, in their study an average value of a_0 was determined from the complete set of notched strength data for the circular hole specimens ($a_0 = 4.9$ mm) and saw-cut specimens ($a_0 = 4$ mm) respectively.

The purpose of the present study is to suggest a simple relation for the characteristic length to improve the accuracy further in evaluation of the notched strength of FRMLs. The analytical results of the notched tensile strength of carbon FRMLs based on Whitney–Nuismer (WN) fracture models are found to be in good agreement with the existing test results [1]. Brief presentation of Whitney–Nuismer's orginal work will be given to enable the reader to grasp the principal rules.

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2. WHITNEY–NUISMER FRACTURE MODELS

Whitney and Nuismer [2, 3] considered that the application of Linear Elastic Fracture Mechanics is questionable since cracks of the types observed in metals do not form in resin–matrix composites under repeated load, and, unlike metals, a positive correlation between the unnotched tensile strength (σ_0) for a composite and its fracture toughness seems to exist (namely the higher the tensile strength, the higher the fracture toughness). Then they proposed a failure model based on the theoretical stress distribution near the notch tip.

Whitney and Nuismer presented two stress fracture criteria known as the point stress criterion (PSC) and the average stress criterion (ASC) for predicting the notched tensile strength of fibre reinforced composites. In the PSC, it is assumed that failure occurs when the stress, σ_y, over some distance, d_0, away from the discontinuity is equal to or greater than the strength of the unnotched laminate (σ_0):

$$\sigma_y(x, 0) = \sigma_0, \quad \text{at } x = R + d_0, \tag{1}$$

where R is the radius of the circular hole.

Similarly, in the ASC, it is assumed that failure occurs when the average stress, σ_y, over some distance, a_0, equals σ_0:

$$(1/a_0) \int_R^{R+a_0} \sigma_y(x, 0)\, \mathrm{d}x = \sigma_0. \tag{2}$$

For an infinite orthotropic plate subjected to a uniform stress, σ_y^∞, applied parallel to the y-axis at infinity, the normal stress, σ_y along the x-axis ahead of the hole can be expressed as [4]

$$\sigma_y(x, 0) = (\sigma_y^\infty/2)\left\{2 + \xi^2 + 3\xi^4 - \left(K_\mathrm{T}^\infty - 3\right)\xi^6\left(5 - 7\xi^2\right)\right\}, \tag{3}$$

where $\xi = R/x$, $x > R$ and the stress concentration factor,

$$K_\mathrm{T}^\infty = 1 + \sqrt{2(\sqrt{E_{xx}/E_{yy}} - \nu_{xy}) + (E_{xx}/G_{xy})}, \tag{4}$$

E_{xx}, E_{yy} and G_{xy} are the axial, transverse, and shear moduli respectively, and ν_{xy} is the major Poisson's ratio for the laminate. Using equation (3) in conjunction with the PSC:

$$\sigma_N^\infty/\sigma_0 = 2\Big\{2 + \Phi_1^2 + 3\Phi_1^4 - \big(K_T^\infty - 3\big)\Phi_1^6\big(5 - 7\Phi_1\big)^2\Big\}^{-1}, \tag{5}$$

where $\Phi_1 = R/(r + d_0)$, (6)

and σ_N^∞ is the notched gross strength of the infinite-width laminate.

Applying the ASC to equation (3):

$$\sigma_N^\infty/\sigma_0 = 2\big\{(1 + \Phi_2)\big[2 + \Phi_2^2 + \big(K_T^\infty - 3\big)\Phi_2^6\big]\big\}^{-1}, \tag{7}$$

with $\Phi_2 = R/(R + a_0)$. (8)

The stress in the vicinity of a crack in an orthotropic laminate of infinite width under uniaxial loading, σ_y^∞, exhibits a singularity at the crack-tip. The exact anisotropic elasticity solution for the normal stress ahead of a crack of length $2c$ in an infinite anisotropic centre cracked plate under uniform tension, σ_y^∞, is given by [5]

$$\sigma_y(x, 0) = \sigma_y^\infty x/\sqrt{x^2 - c^2}, \quad x > c. \tag{9}$$

It should be noted that the stress fields in the close neighbourhood of the crack-tip are functions of material properties and the normal stress along the crack plane ahead of the crack-tip is independent of material properties [6]. Because of the singularity of the crack-tip, the concept of a stress concentration factor is replaced by a stress intensity factor. For uniaxial tension (mode I), this factor is defined as:

$$K_I = \sigma_y^\infty\sqrt{\pi c}. \tag{10}$$

The stress distribution, equation (10), in terms of the stress intensity factor is given by

$$\sigma_y(x, 0) = K_I x/\sqrt{\pi c(x^2 - c^2)}, \quad x > c. \tag{11}$$

Applying the PSC in conjunction with equation (9) or (11), the notch sensitivity of an infinite laminated plate with a centre crack becomes:

$$\sigma_N^\infty/\sigma_0 = \sqrt{1 - \Phi_3^2}, \tag{12}$$

with $\Phi_3 = \{c/(c + d_0)\}$. (13)

From equations (10) and (12), the critical stress intensity factor, K_Q, becomes

$$K_Q = \sigma_0\sqrt{\pi c(1 - \Phi_3^2)}. \tag{14}$$

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Applying the ASC in conjunction with equation (9) or (11), the notch sensitivity ratio results as:

$$\sigma_{\mathrm{N}}^{\infty}/\sigma_0 = \sqrt{(1-\Phi_4)/(1+\Phi_4)}, \tag{15}$$

with $\Phi_4 = \{c/(c+a_0)\}$. (16)

Equations (5), (7), (12) and (15) are also given in Ref. [4].

From equations (10) and (15), the critical stress intensity factor, K_{Q}, becomes

$$K_{\mathrm{Q}} = \sigma_0\sqrt{\pi c(1-\Phi_4)/(1+\Phi_4)}. \tag{17}$$

In both the equations (14) and (17), the expected limit of $K_{\mathrm{Q}} = 0$ for vanishingly small ($c \ll d_0$ or a_0) crack lengths is reached, while for large crack lengths ($c \gg d_0$ or a_0), K_{Q} asymptotically approaches a constant value ($K_{\mathrm{Q}\infty}$). For the point and average stress criteria, these asymptotic values are respectively

$$K_{\mathrm{Q}\infty} = \sigma_0\sqrt{2\pi d_0}, \tag{18}$$

$$K_{\mathrm{Q}\infty} = \sigma_0\sqrt{\pi a_0/2}. \tag{19}$$

The above discussed Whitney–Nuismer fracture models for laminated composites with a circular hole and a crack, are two different cases. These fracture models are similar to the inherent flaw model of Waddoups *et al*. [7] in which their model assumes that near the hole there are regions of intense energy of length transverse to the loading direction and the intense energy regions are considered as cracks. Hence, a characteristic size (d_0 or a_0) in each case is considered to be a damage zone immediately ahead of the hole or crack. If the performance of a laminate is affected by the stress concentration, it is defined as 'notch sensitive'. If it is affected by the change of opening shape with a same opening length, the laminate is defined as 'notch shape sensitive'. The fracture data existing on sevaral composite laminates indicates that most laminates are 'notched shape sensitive'. The damage zone immdiately ahead of the hole is expected to be different from that of a crack. It can be observed from equations (5), (7), (14) and (17) that the strength of a laminate decreases with the hole size and the critical intensity factor increases with the crack size. Whitney–Nuismer fracture models for laminated composites with a circular hole or with a crack, are based on the unnotched tensile stregth (σ_0) and a characteristic dimension (d_0 or a_0). In each case, the notched tensile strength reduces with the size of discontinuity (hole or crack). Since the behaviour of fracture strength is assumed to be similar for laminated composites with a circular hole or with a crack, it is more appropriate to define the fracture toughness (K_{Q_∞}) in terms of σ_0 and the characteristic dimension (d_0 or a_0) as given by equations (18) and (19).

It is evident from the fracture data that the characteristic dimension (d_0 or a_0) increases and the notched tensile strength ($\sigma_{\mathrm{N}}^{\infty}$) decreases with hole/crack size. This implies that $K_{\mathrm{Q}\infty}$ in equations (18) and (19) increases with the characteristic dimension. Hence it is more appropriate to have a relationship between $K_{\mathrm{Q}\infty}$ and

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σ_N^∞ so that for any notch size the corresponding characteristic dimension can be obtained directly from the relation. Following the two parameter fracture criterion, developed by Newman [8] for metallic materials, a linear relation between $K_{Q\infty}$ and σ_N^∞ is proposed as

$$K_{Q\infty} = K_F\left\{1 - m(\sigma_N^\infty/\sigma_0)\right\}. \tag{20}$$

The parameters K_F and m in equation (20) are determined by a least–square curve fit to the data of $K_{Q\infty}$, σ_N^∞ and σ_0. The nondimensional value of m in general is greater than zero and less than unity. Whenever m is found to be greater than unity, the parameter m has to be truncated to 1.0 and the other unknown parameter K_F in equation (20) is obtained by a least–squares fit to the fracture data. If m is found to be less than zero, the parameter m has to be truncated to zero and the average of $K_{Q\infty}$ values from the fracture data yields the parameter K_F.

When the point stress criterion and the average stress criterion are used, one gets two sets of equations. Since the characteristic dimensions d_0 or a_0 for the two fracture models are different, the values of K_F and m in equation (20) become different. Hence, the parameters K_F and m in equation (20) have to be determined separately for the laminated composites having circular holes and cracks. The characteristic length in equation (20) represents a quadratic polynomial in terms of the notched strength of a laminate. The variation of the characteristic length depends on the value of the parameter m in equation (20). When m is equal to zero, the characteristic length is independent of the notch size.

Evaluation of characteristic length from a simple relation (20) for notched tensile strength of composite laminates through Whitney–Nuismer fracture models, has several advantages compared to the exisiting empirical relations. In Whitney–Nuismer fracture models, it is assumed that the characteristic dimension has the same values for all notch sizes, which may not be valid for all materials. Pipes *et al*. [9, 10] extended the Whitney–Nuismer (WN) fracture model and introduced a three-parameter notched strength model in which the characteristic length,

$$d_0 = C_p(R/R_0)^{m_p}, \tag{21}$$

is assumed as a function of the hole size (R), reference radius (R_0), notch sensitivity factor ($C_p > 0$), and exponential parameter ($0 < m_p < 1$).

Kim *et al*. [11] expressed the possibility of varying the notch sensitivity factor (C_p) with the selection of the reference radius (R_0), and considered the characteristic length,

$$d_0 = C_k(2R/W)^{m_k}, \tag{22}$$

in the PSC as a function of hole diameter ($2R$), specimen width (W), notch sensitivity factor (C_k), and exponential parameter (m_k). Based on the test results, Kim *et al*. [11] suggested a relation for d_0, given by equation (22), which is a function of hole size and width of the plate. The constants in equation (22) presented by them are found to vary with the width of the plate and hence introducing the width (W) in equation (22) in place of the reference radius (R_0) in equation (21) has no additional benefits. When the parameters change for each width configuration, calculating the constants either in equation (21) or in equation (22) may not be useful for designers in designing a component in advance because they have to conduct tests for the intended diameter and width to get the actual notched tensile strength value. Instead, they can perform the test directly for the intended diameter and width to get the actual notched tensile strength value. In such a case, analytical modelling may not be useful to designers. Kim *et al*. [11] also tried to correlate the notched strength with the characteristic length using a linear relationship. The constants in the relation are also found to vary with the width of the plate. Such a relation also will not be directly useful.

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3. RESULTS AND DISCUSSION

The Whitney–Nuismer (WN) fracture models known as the point stress criterion and the average stress criterion are applied to correlate the residual strength of FRMLs with the size of various saw-cut/cracks and circular holes of various sizes [1]. A value of 3 for K_T^∞ as obtained in Ref. [1] was used for circular hole specimens. The notched tensile strength of an infinite plate (σ_N^∞) is obtained by multiplying the experimental notched strength of a finite width plate (σ_N) by a correction factor (K_T/K_T^∞) as defined in Ref. [12]:

$$K_T^\infty/K_T = \alpha + \beta^3(K_T^\infty - 3)(1-\beta)/2, \tag{23}$$

where $\alpha = 3(1 - D/W)/\{2 + (1 - D/W)^3]$, $\beta = (\sqrt{9-8\alpha} - 1)/2$, $D = 2R$, is the diameter of holes and W is the width of the specimens. Substituting the values of σ_N^∞ and σ_0 in equations (5) and (7) and solving them using Newton–Raphson iterative scheme, the characteristic lengths, d_0 and a_0 corresponding to the dimension of the circular hole is determined. The asymptotic values of $K_{Q\infty}$ are obtained by using the values of σ_0, d_0 and a_0 in equations (18) and (19). The parameters K_F and m in equation (20) are determined through a least–square curve fit from the generated data. Similarly, the tensile strength of an infinite saw-cut specimen (σ_N^∞) is obtained by multiplying the experimental tensile strength of a finite width plate (σ_N) by a finite width correction function [8];

$$Y = \sqrt{\sec(\pi c/w)}. \tag{24}$$

Substituting σ_N^∞ and σ_0 in equations (12) and (15), the charactistic lengths d_0 and a_0 corresponding to the dimension of the saw-cut, $2c$, are determined from

$$d_0 = c\left[\left\{1 - (\sigma_N^\infty/\sigma_0)^2\right\}^{-1} - 1\right], \tag{25}$$

$$a_0 = c\left[\left\{1 + (\sigma_N^\infty/\sigma_0)^2\right\}\left\{1 - (\sigma_N^\infty/\sigma_0)^2\right\}^{-1} - 1\right]. \tag{26}$$

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The asymptotic values of $K_{Q\infty}$ are obtained by using the values of σ_0, d_0 and a_0 in equations (18) and (19). The parameters K_F and m in equation (20) for the saw-cut specimens are determined through a least–square curve fit from the generated data. In order to distinguish the parameters K_F and m in PSC and ASC for circular hole specimens and saw-cut specimens, they are designated by K_{F_i} and m_i $(i = 1, 2, 3, 4)$. Tables 1 and 2 give the parameters K_F and m from the notched tensile specimens made of carbon FRMLs, ARALL and GLARE. Figures 1–3 show the plot of $K_{Q\infty}$ *versus* the ratio of the notched strength of infinite-width plate (σ_N^∞) to the unnotched strength (σ_0) for carbon FRMLs, ARALL and GLARE. The generated data in Tables 1 and 2 on $K_{Q\infty}$ and σ_N^∞/σ_0 are also marked in Figs 1–3. Since the characteristic dimensions (d_0 and a_0) are different for the two fracture models, the values of K_F and m in equation (20) obtained through a least–square curve fit and presented in Tables 1 and 2, are different. It can be seen in Figs 1–3 that two lines correspond to PSC and ASC for center circular hole specimens, and the other two lines correspond to PSC and ASC for saw-cut/cracked specimens. All the generated data falls close to the fitted lines and validates the assumed simple relation, equation (20), for $K_{Q\infty}$ and σ_N^∞/σ_0. This relation can be used for the characteristic length while evaluating the tensile fracture strength of composite laminates containing circular holes or saw-cuts through Whitney–Nuismer fracture models. The linear relation (20) assumed for the PSC and ASC represent the characteristic length as a quadratic polynomial in terms of σ_N^∞/σ_0.

For centre circular hole specimens, the characteristic dimensions d_0 and a_0 in terms of σ_N^∞/σ_0 are expressed in the form

$$d_0 = \gamma_1/2\pi, \tag{27}$$

$$a_0 = 2\gamma_2/\pi. \tag{28}$$

For saw-cut/cracked specimens, d_0 and a_0 are expressed in the form

$$d_0 = \gamma_3/2\pi, \tag{29}$$

$$a_0 = 2\gamma_4/\pi, \tag{30}$$

γ_i's ($i = 1$ to 4) in equations (27)–(30) are defined by

$$\gamma_i = \left(K_{Fi}/\sigma_0\right)^2 \left\{1 - m_i(\sigma_N^\infty/\sigma_0)\right\}^2. \tag{31}$$

For the specified hole size or crack size, the notched tensile strength σ_N^∞ can be determined from any one of the equations (5), (7), (12) or (15), which are functions of Φ_i. Equations (6), (8), (13) and (16) give Φ_i's as function of the hole or crack size, and the characteristic length, d_0 or a_0. Since the characteristic length is expressed as a quadratic polynomial in terms of σ_N^∞/σ_0, Φ_i's are written in terms of σ_N^∞/σ_0 by using equations (27)–(30) in equations (6), (8), (13) and (16). These are

$$\Phi_1 = \{1 + \gamma_1/(2\pi R)\}^{-1}, \tag{32}$$

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Table 1.
Fracture toughness parameters K_F and m in equation (20) for FRMLs with circular holes

Width (mm)	D $(= 2R)$ (mm)	K_T	σ_N (MPa) Test [1]	$\sigma_N^{\infty}/\sigma_0$	Point stress criterion (PSC)		Average stress criterion (ASC)	
					d_0 (mm)	$K_{Q\infty}$ (MPa$\sqrt{\mathrm{m}}$)	a_0 (mm)	$K_{Q\infty}$ (MPa$\sqrt{\mathrm{m}}$)
Carbon FRML*: $\sigma_0 = 883.4$ MPa; $K_{F1} = 182.2$ MPa$\sqrt{m}$, $m_1 = 0.917$; $K_{F2} = 118.3$ MPa$\sqrt{\mathrm{m}}$, $m_2 = 0.713$								
90.04	10.26	3.04	488.25	0.560	1.63	89.49	4.20	71.75
89.94	19.92	3.18	404.34	0.484	2.01	99.35	4.72	76.10
90.02	30.26	3.45	344.21	0.449	2.30	106.09	5.18	79.66
90.10	40.08	3.91	292.30	0.431	2.57	112.27	5.69	83.52
ARALL-2**: $\sigma_0 = 717$ MPa; $K_{F1} = 131.4$ MPa$\sqrt{m}$, $m_1 = 1$; $K_{F2} = 86.39$ MPa$\sqrt{m}$, $m_2 = 0.827$								
100	10	3.03	358.83	0.506	1.17	61.39	2.81	47.61
100	10	3.03	363.33	0.512	1.21	62.62	2.94	48.75
100	25	3.23	302.90	0.455	2.00	80.44	4.55	60.60
100	50	4.25	196.98	0.389	1.81	76.41	3.83	55.60
GLARE-2***: $\sigma_0 = 1230$ MPa; $K_{F1} = 276.2$ MPa$\sqrt{m}$, $m_1 = 1$; $K_{F2} = 197.8$ MPa$\sqrt{m}$, $m_2 = 0.943$								
100	10	3.03	669.68	0.550	1.51	119.78	3.83	95.42
100	25	3.23	526.60	0.461	2.11	141.60	4.82	107.04
99.9	50	4.25	348.32	0.402	2.21	145.01	4.74	106.18
99.7	75	8.13	187.58	0.414	3.91	192.88	8.50	142.13

* 2/1 Lay-up of 2024-T3 + UD Carbon prepreg.
** 2/1 Lay-up of 2024-T3 + UD Aramid prepreg.
*** 2/1 Lay-up of 2024-T3 + UD R-glass prepreg.

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Table 2.
Fracture toughness parameters K_F and m in equation (20) for FRMLs with saw-cuts

Width (mm)	$2c$ (mm)	Y	σ_N (MPa) Test [1]	σ_N^∞/σ_0	Point stress criterion (PSC) $K_{Q\infty}$ (MPa$\sqrt{m}$)	Average stress criterion (ASC) $K_{Q\infty}$ (MPa$\sqrt{m}$)
Carbon FRML: $\sigma_0 = 883.4$ MPa; $K_{F3} = 88.43$ MPa$\sqrt{m}$, $m_3 = 0.537$; $K_{F4} = 87.06$ MPa$\sqrt{m}$, $m_4 = 0.460$						
90.02	10.29	1.0081	422.34	0.482	59.71	61.78
90.10	10.03	1.0077	439.41	0.501	61.87	64.23
89.94	19.97	1.0315	361.46	0.422	71.04	72.84
89.94	19.81	1.0310	370.16	0.432	72.70	74.65
90.00	30.08	1.0750	288.63	0.351	70.84	72.03
90.12	29.45	1.0714	306.13	0.371	74.56	75.98
90.00	30.04	1.0748	304.81	0.371	75.20	76.63
90.12	29.41	1.0712	296.25	0.359	71.81	73.09
90.04	29.38	1.0712	302.78	0.367	73.54	74.91
90.14	39.99	1.1419	240.42	0.311	71.47	72.39
90.18	40.21	1.1436	235.73	0.305	70.27	71.15
90.10	40.07	1.1428	237.51	0.307	70.66	71.55
ARALL-2: $\sigma_0 = 717$ MPa; $K_{F3} = 57.38$ MPa$\sqrt{m}$, $m_3 = 0$; $K_{F4} = 58.15$ MPa$\sqrt{m}$, $m_4 = 0$						
100	25	1.0404	271.13	0.366	58.97	60.06
100	50	1.1892	163.31	0.252	55.78	56.24
GLARE-2: $\sigma_0 = 1230$ MPa; $K_{F3} = 105.09$ MPa$\sqrt{m}$, $m_3 = 0$; $K_{F4} = 106.91$ MPa$\sqrt{m}$, $m_4 = 0$						
99.8	25	1.0406	478.22	0.405	105.39	107.83
100	50	1.1892	304.00	0.294	104.79	106.00

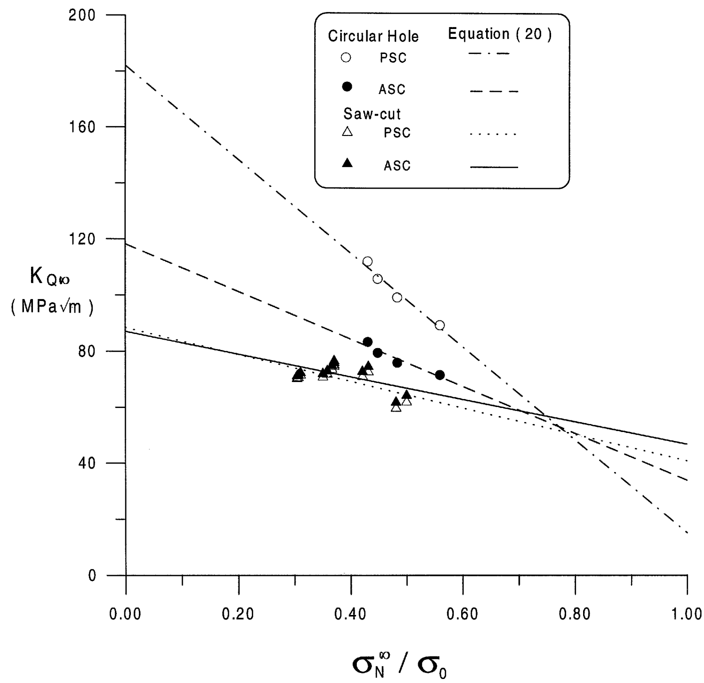

Figure 1. Plot of $K_{Q\infty}$ versus σ_N^∞/σ_0 for carbon FRML laminates.

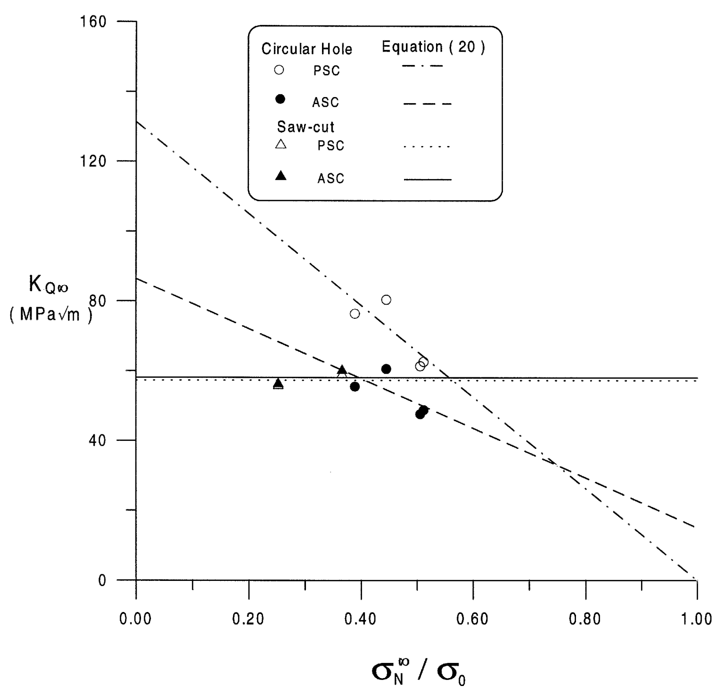

Figure 2. Plot of $K_{Q\infty}$ versus σ_N^∞/σ_0 for ARALL-2 laminates.

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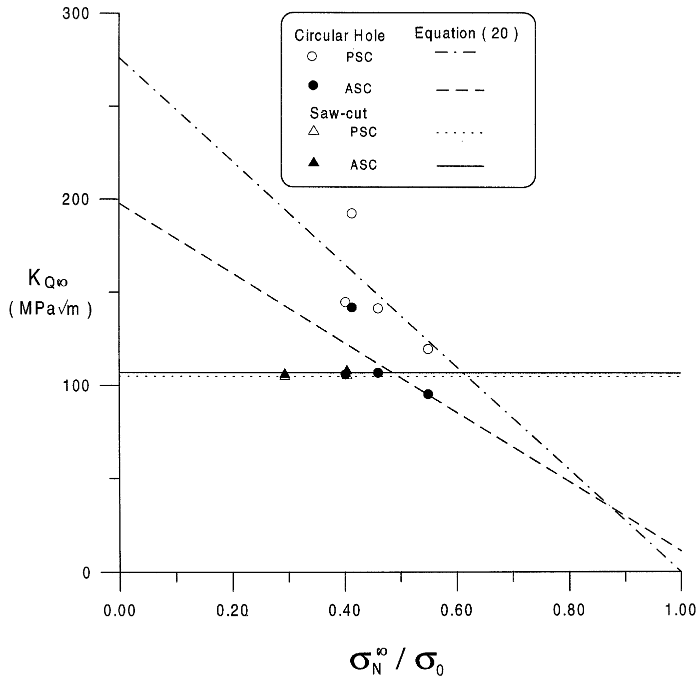

Figure 3. Plot of $K_{Q\infty}$ versus σ_N^∞/σ_0 for GLARE-2 laminates.

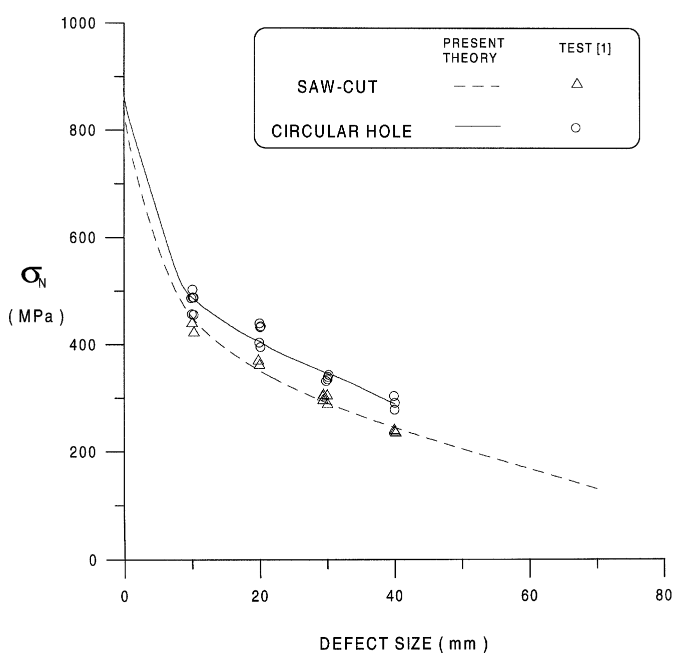

Figure 4. Notched strength curves for carbon FRML laminates ($W = 100$ mm) having saw-cut and circular hole based on point stress criterion.

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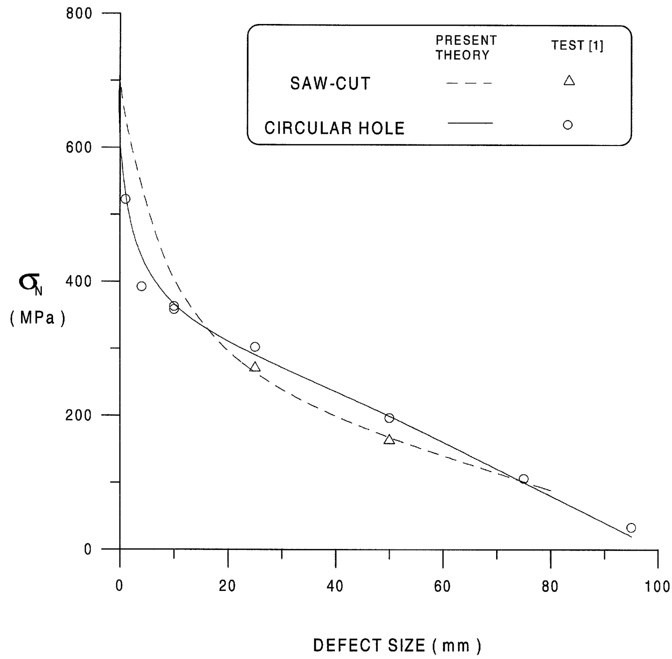

Figure 5. Notched strength curves for ARALL-2 laminates ($W = 100$ mm) having saw-cut and circular hole based on point stress criterion.

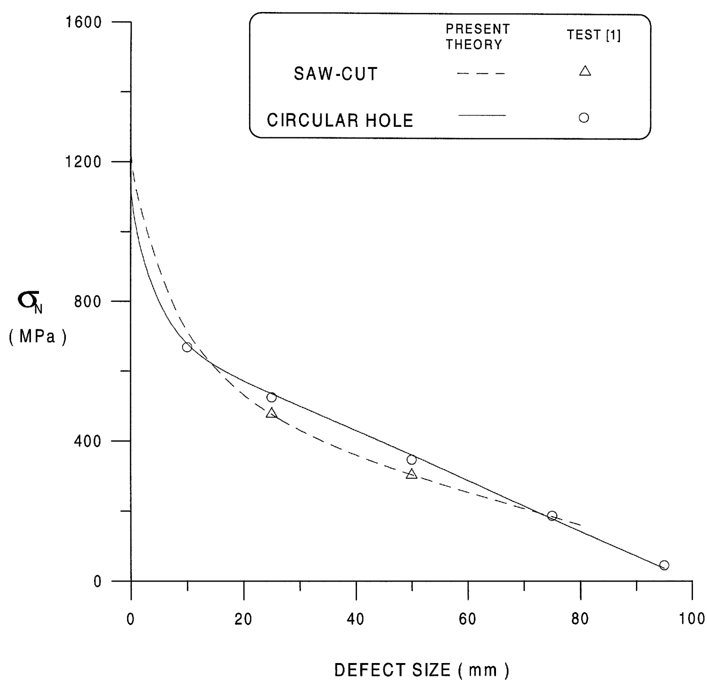

Figure 6. Notched strength curves for GLARE-2 laminates ($W = 100$ mm) having saw-cut and circular hole based on point stress criterion.

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Table 3.
Comparison of notched tensile strength, σ_N (MPa) of carbon FRMLs

Width W (mm)	Notch dimension $2R$ or $2c$ (mm)	Test [1] σ_N (MPa)	ASC [1] σ_N (MPa)	ASC [1] Relative error (%)	Present study PSC σ_N (MPa)	Present study PSC Relative error (%)	Present study ASC σ_N (MPa)	Present study ASC Relative error (%)
CIRCULAR HOLE SPECIMENS								
90.04	10.26	488.25	509.68	−4.4	486.19	0.4	486.36	0.4
90.06	10.08	503.58	512.45	−1.8	488.16	3.1	488.36	3.0
90.24	9.96	457.30	514.36	−12.5	489.52	−7.1	489.74	−7.1
90.22	9.88	487.49	515.62	−5.8	490.42	−0.6	490.64	−0.7
89.90	10.12	488.81	511.81	−4.7	487.70	0.2	487.89	0.2
89.94	10.23	456.32	510.12	−11.8	486.50	−6.6	486.67	−6.7
89.94	19.92	404.34	408.08	−0.9	407.41	−0.8	407.24	−0.7
90.16	20.16	435.12	406.37	6.6	405.97	6.7	405.80	6.7
90.10	20.11	396.31	406.71	−2.6	406.26	−2.5	406.09	−2.5
90.20	19.99	440.38	407.68	7.4	407.10	7.6	406.93	7.6
89.96	20.07	433.45	406.88	6.1	406.39	6.2	406.22	6.3
90.02	30.26	344.21	340.14	1.2	345.60	−0.4	345.55	−0.4
89.90	30.11	340.62	340.89	−0.1	346.30	−1.7	346.26	−1.7
90.00	30.07	335.99	314.24	−1.6	346.65	−3.2	346.60	−3.2
90.00	29.83	332.52	342.67	−3.1	348.01	−4.7	347.97	−4.6
90.10	40.08	292.30	284.05	2.8	290.40	0.7	290.45	0.6
90.10	39.94	304.91	284.84	6.6	291.19	4.5	291.24	4.5
90.10	40.06	279.44	284.16	−1.7	290.51	−4.0	290.57	−4.0
SAW-CUT SPECIMENS								
90.02	10.29	422.34	463.61	−9.8	447.64	−6.0	447.06	−5.9
90.10	10.03	439.41	468.08	−6.5	451.33	−2.7	450.70	−2.9
89.94	19.97	361.46	349.84	3.2	352.06	2.6	352.17	2.6
89.94	19.81	370.16	351.19	5.1	353.23	4.6	353.34	4.5
90.00	30.08	288.63	281.53	2.5	291.40	−1.0	291.49	−1.0
90.12	29.45	306.13	285.12	6.9	294.68	3.7	294.78	3.7
90.00	30.04	304.81	281.75	7.6	291.60	4.3	291.69	4.3
90.12	29.41	296.25	285.35	3.7	294.89	0.5	294.99	0.4
90.04	29.38	302.78	285.48	5.7	295.01	2.6	295.11	2.5
90.14	39.99	240.42	233.27	2.9	246.17	−2.4	246.15	−2.4
90.18	40.21	235.73	232.35	1.4	245.28	−4.1	245.26	−4.0
90.10	40.07	237.51	232.90	1.9	245.80	−3.5	245.79	−3.5

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Table 4.
Comparison of notched tensile strength, σ_N (MPa) of ARALL-2

Width W (mm)	Notch dimension $2R$ or $2c$ (mm)	Test [1] σ_N (MPa)	ASC [1] σ_N (MPa)	ASC [1] Relative error (%)	Present study PSC σ_N (MPa)	PSC Relative error (%)	ASC σ_N (MPa)	ASC Relative error (%)
CIRCULAR HOLE SPECIMENS								
100	1	522.62	653.37	−25.0	532.25	−1.8	540.90	−3.5
100	4	392.59	532.59	−35.7	437.04	−11.3	438.35	−11.7
100	10	358.83	418.02	−16.5	366.54	−2.1	366.32	−2.1
100	10	363.33	418.02	−15.1	366.54	−0.9	366.62	−0.8
100	25	302.90	308.13	−1.7	290.93	4.0	290.90	4.0
100	50	196.98	204.25	−3.7	199.38	−1.2	199.53	−1.3
100	75	106.17	101.77	4.1	100.38	5.5	100.48	5.4
100	95	33.43	19.99	40.2	19.80	40.8	19.82	40.7
SAW-CUT SPECIMENS								
100	25	271.13	275.23	−1.5	264.51	2.4	263.61	2.7
100	50	163.31	176.45	−8.0	167.76	−2.7	168.49	−3.2

Table 5.
Comparison of notched tensile strength, σ_N (MPa) of GLARE-2

Width W (mm)	Notch dimension $2R$ or $2c$ (mm)	Test [1] σ_N (MPa)	ASC [1] σ_N (MPa)	ASC [1] Relative error (%)	Present study PSC σ_N (MPa)	PSC Relative error (%)	ASC σ_N (MPa)	ASC Relative error (%)
CIRCULAR HOLE SPECIMENS								
100	10	669.68	717.11	−7.1	678.77	−1.4	669.17	0.1
100	25	526.60	528.59	−0.4	536.42	−1.9	534.69	−1.5
99.9	50	348.32	350.10	−0.5	361.81	−3.9	362.85	−4.2
99.7	75	187.58	173.05	7.8	178.74	4.7	179.55	4.3
100.2	95	45.48	35.59	21.7	36.68	19.4	36.86	19.0
SAW-CUT SPECIMENS								
99.8	25	478.22	439.01	8.2	477.03	0.2	474.82	0.7
100	50	304.00	281.50	7.4	304.81	−0.3	306.40	−0.8

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Table 6.
A comparison between the experimental and estimated notched tensile strength values for woven glass/epoxy laminates. ($W = 5D$)

D $(= 2R)$ (mm)	Notched strength, σ_N (MPa) Test [13]	Present theory: Point stress criterion (PSC) σ_N (MPa)	PSC Relative error (%)	Average stress criterion (ASC) σ_N (MPa)	ASC Relative error (%)
Glass-1					
Lay-up: $[0/0/45/0/0/-45/90/90]_S$; $\sigma_0 = 467.1$ MPa; $K_{F1} = 74.49$ MPa$\sqrt{m}$; $m_1 = 1.0$; $K_{F2} = 54.35$ MPa$\sqrt{m}$; $m_2 = 0.909$					
10	221.9	219.3	1.2	220.1	0.8
8	227.6	229.2	−0.7	229.6	−0.9
5	250.0	250.9	−0.4	250.3	−0.1
3	273.7	274.7	−0.4	273.3	0.2
Glass-2					
Lay-up: $[0/45/0/90/0/-45/90/0]_S$; $\sigma_0 = 442.5$ MPa; $K_{F1} = 71.42$ MPa$\sqrt{m}$; $m_1 = 0.870$; $K_{F2} = 48.53$ MPa$\sqrt{m}$; $m_2 = 0.682$					
10	218.0	220.0	−0.9	220.1	−1.0
8	233.3	230.9	1.1	230.8	1.1
5	255.5	254.7	0.3	254.5	0.4
3	279.4	280.7	−0.5	280.8	−0.5
Glass-3					
Lay-up: $[0/0/45/45/-45/-45/90/90]_S$; $\sigma_0 = 307.7$ MPa; $K_{F1} = 37.84$ MPa$\sqrt{m}$; $m_1 = 0.351$; $\mathrm{K}_{F2} = 25.15$ MPa$\sqrt{m}$; $m_2 = 0$					
10	163.1	163.6	−0.3	166.5	−2.1
8	175.2	174.8	0.2	177.2	−1.1
5	201.6	201.0	0.3	200.8	0.4
3	229.8	230.2	−0.2	225.5	1.9

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Table 7a.
A comparison between the experimental and estimated notched tensile strength values for woven carbon/epoxy laminates having drilled holes. ($W = 25.4$ mm)

$D\ (= 2R)$ (mm)	Notched strength, σ_N (MPa) Test [14]	Present theory: Point stress criterion (PSC) σ_N (MPa)	Relative error (%)	Average stress criterion (ASC) σ_N (MPa)	Relative error (%)
Lay-up: $[0]_{4S}$; $\sigma_0 = 655.5$ MPa; $K_{F1} = 55.7$ MPa$\sqrt{m}$; $m_1 = 0.308$; $K_{F2} = 43.9$ MPa$\sqrt{m}$; $m_2 = 0$					
3.15	462.6	464.5	−0.4	446.5	3.5
6.53	340.3	335.1	1.5	339.5	0.2
9.5	255.9	258.4	−1.0	272.5	−6.5
Lay-up: $[45/0]_{2S}$; $\sigma_0 = 501.4$ MPa; $K_{F1} = 59.1$ MPa$\sqrt{m}$; $m_1 = 0.508$; $K_{F2} = 36.7$ MPa$\sqrt{m}$; $m_2 = 0.068$					
3.18	345.9	347.8	−0.6	347.6	−0.5
6.35	278.0	274.2	1.4	274.3	1.3
9.53	218.7	220.4	−0.8	220.5	−0.8
Lay-up: $[0/45]_{2S}$; $\sigma_0 = 454.7$ MPa; $K_{F1} = 51.2$ MPa$\sqrt{m}$; $m_1 = 0.403$; $K_{F2} = 33.5$ MPa$\sqrt{m}$; $m_2 = 0$					
3.07	329.1	329.2	0	326.4	0.8
6.32	255.3	255.1	0.1	255.9	−0.3
9.53	203.1	203.2	0	204.8	−0.8

Table 7b.
A comparison between the experimental and estimated notched tensile strength values for woven kevlar/epoxy laminates having drilled holes. ($W = 25.4$ mm)

$D\ (= 2R)$ (mm)	Notched strength, σ_N (MPa) Test [14]	Present theory: Point stress criterion (PSC) σ_N (MPa)	Relative error (%)	Average stress criterion (ASC) σ_N (MPa)	Relative error (%)
Lay-up: $[0]_{4S}$; $\sigma_0 = 503.7$ MPa; $K_{F1} = 89.0$ MPa$\sqrt{m}$; $m_1 = 0.789$; $K_{F2} = 65.0$ MPa$\sqrt{m}$; $m_2 = 0.446$					
3.33	378.8	377.2	0.4	377.3	0.4
6.45	303.9	315.2	−3.7	314.3	−3.4
9.63	269.5	261.0	3.1	261.8	2.9
Lay-up: $[45/0]_{2S}$; $\sigma_0 = 384.1$ MPa; $K_{F1} = 41.5$ MPa$\sqrt{m}$; $m_1 = 0.243$; $K_{F2} = 30.32$ MPa$\sqrt{m}$; $m_2 = 0$					
3.12	290.5	291	−0.2	283.4	−3.0
6.53	220.8	219.5	0.6	222.0	−0.6
9.6	174.4	175.0	−0.4	179.7	2.5
Lay-up: $[0/45]_{2S}$; $\sigma_0 = 369.2$ MPa; $K_{F1} = 46.9$ MPa$\sqrt{m}$; $m_1 = 0.250$; $K_{F2} = 35.7$ MPa$\sqrt{m}$; $m_2 = 0$					
3.15	302.2	301.4	0.3	293.9	2.7
6.58	230.6	233.6	−1.3	236.6	−2.6
9.63	188.9	187.1	0.9	193.2	−2.3

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Table 7c.
A comparison between the experimental and estimated notched tensile strength values for woven carbon-kevlar/epoxy laminates having drilled holes. ($W = 25.4$ mm)

D $(= 2R)$ (mm)	Notched strength σ_N (MPa) Test [14]	Present theory: Point stress criterion (PSC) σ_N (MPa)	Relative error (%)	Average stress criterion (ASC) σ_N (MPa)	Relative error (%)
Lay-up: $[45/0]_{2S}$; $\sigma_0 = 355.4$ MPa; $K_{F1} = 39.7$ MPa$\sqrt{m}$; $m_1 = 0$; $K_{F2} = 42.0$ MPa$\sqrt{m}$; $m_2 = 0.222$					
3.18	307.1	304.0	1.0	282.5	8.0
6.5	232.2	238.9	−2.9	232.2	0
9.55	192.0	189.9	1.1	192.0	0
Lay-up: $[0/45]_{2S}$; $\sigma_0 = 326.4$ MPa; $K_{F1} = 39.9$ MPa$\sqrt{m}$; $m_1 = 0$; $K_{F2} = 36.8$ MPa$\sqrt{m}$; $m_2 = 0$					
3.18	291.4	287.9	1.2	271.9	6.7
6.45	233.2	232.1	0.5	226.2	3.0
9.58	178.5	184.9	−3.6	186.1	−4.2

Table 8.
A comparison between the experimental and estimated notched tensile strength values for carbon T300/914C laminates having center crack

W (mm)	D $(= 2R)$ (mm)	Notched strength, σ_N (MPa) Test [15]	Present theory: Point stress criterion (PSC) σ_N (MPa)	Relative error (%)	Average stress criterion (ASC) σ_N (MPa)	Relative error (%)
Lay-up: $[(\pm45/0/90)_3/0/90/\pm45]_S$; $\sigma_0 = 548.0$ MPa; $K_{F3} = 72.9$ MPa$\sqrt{m}$; $m_3 = 0.611$; $K_{F4} = 69.2$ MPa$\sqrt{m}$; $m_4 = 0.488$						
36.2	6	355.7	356.3	−0.2	356.3	−0.2
36.3	10	304.7	305.3	−0.2	305.3	−0.2
48.1	8	334.3	333.1	0.4	333.1	0.4
60.2	10	350.7	314.9	10.2	314.9	10.2

$$\Phi_2 = \{1 + 2\gamma_2/(\pi R)\}^{-1}, \tag{33}$$

$$\Phi_3 = \{1 + \gamma_3/(2\pi c)\}^{-1}, \tag{34}$$

$$\Phi_4 = \{1 + 2\gamma_4/(\pi c)\}^{-1}. \tag{35}$$

Using equation (32) in (5) and eliminating Φ_1, a nonlinear equation in terms of σ_N^∞/σ_0 and R is obtained for circular hole specimens based on PSC. Using equation (33) in (7) and eliminating Φ_2, another nonlinear equation in terms of σ_N^∞/σ_0 and R is obtained for circular hole specimens based on ASC. For any specified value of R, σ_N^∞/σ_0 can be determined by solving these equations separately through

Table 9.
A comparison between the experimental and estimated notched tensile strength values for carbon T300/N5208 laminates having center crack

W (mm)	$D\ (=2R)$ (mm)	Notched strength σ_N (MPa) Test [15]	Present Theory: Point stress criterion (PSC) σ_N (MPa)	Relative error (%)	Average stress criterion (ASC) σ_N (MPa)	Relative error (%)
Lay-up: $[0/90]_{4S}$; $\sigma_0 = 636.0$ MPa; $K_{F3} = 148.7$ MPa$\sqrt{m}$; $m_3 = 1.0$; $K_{F4} = 155.5$ MPa$\sqrt{m}$; $m_4 = 1.0$						
25.1	2.8	471.9	459.6	2.6	453.9	3.8
25.1	7.8	321.2	380.1	−18.3	377.4	−17.5
50.5	15.2	322.5	340.4	−5.6	339.7	−5.3
76.0	25.2	319.3	303.4	5.0	304.0	4.8
Lay-up: $[0/\pm45/90]_{2S}$; $\sigma_0 = 494.0$ MPa; $K_{F3} = 64.8$ MPa$\sqrt{m}$; $m_3 = 0.222$; $K_{F4} = 60.5$ MPa$\sqrt{m}$; $m_4 = 0.022$						
25.3	2.8	433.2	439.5	−1.5	429.1	1.0
25.4	7.8	316.7	344.8	−8.9	342.3	−8.1
50.6	15.2	287.0	287.0	0	287.0	0
76.2	25.2	239.1	239.1	0	239.1	0

Newton–Raphson iterative scheme. The notched strength of a finite width plate is then obtained by multiplying the determined σ_N^∞ with K_T^∞/K_T for both the cases.

For saw-cut specimens, using equation (34) in (12) and eliminating Φ_3, a nonlinear equation in terms of σ_N^∞/σ_0 and c is obtained for PSC. Using equation (35) in (15) and eliminating Φ_4, another nonlinear equation in terms of σ_N^∞/σ_0 and c is obtained for ASC. For any specified value of c, σ_N^∞/σ_0 can be determined by solving these equations separately through Newton–Raphson iterative scheme. The tensile strength of finite width plate is then obtained by dividing σ_N^∞ with Y for both the cases. Tables 3 to 5 give comparison of analytical and experimental results. The relative error in the notched tensile strengh using the ASC, is found to be more by considering the average value of the characteristic length (a_0) as suggested in Ref. [1] for circular hole specimens and saw-cut specimens. The analytical results obtained from PSC are very close to those obtained from ASC. Figures 4 to 6 show the variation of notched tensile strength, σ_N, with the defect size for different FRMLs using the PSC. The analytical results obtained in the present study are close to the test results except for the large notch size, $2R\ (= 0.95\ W)$. It is noted from Ref. [1] that the large notched specimens are tending to act like dual tensile tests, which resulted in high values of strength. This may be the reason for the large disparity between analytical and test results for large notch size.

In order to examine the validity of the present model, fracture data existing on different material systems have also considered. Table 6 gives a good comparison between the experimental [13] and estimated notched tensile strength values for woven glass/epoxy laminates. Tables 7a to 7c show a good comparison between

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the experimental results [14] and estimated values for three woven fabric reinforced composite systems (carbon, kevlar and carbon-kevlar in epoxy matrix). Tables 8 and 9 show the comparison of results for carbon/epoxy composite laminates.

4. CONCLUDING REMARKS

The modifications in Whitney–Nuismer fracture criteria to estimate the characteristic length for evaluation of notched tensile strength of laminates made in this paper have several advantages. In the Whitney–Nuismer fracture model, the characteristic dimension is assumed to have the same value for all notch sizes, which may be applicable for few materials. The linear relationship between $K_{Q\infty}$ and σ_N^∞ in equation (20) assumed for the point stress criterion and the average stress criterion represent the characteristic dimension as a quadratic polynomial in terms of σ_N^∞. The characteristic dimension variation depends upon the value of the parameter, m. When m is zero, the characteristic dimension is independent of the notch size. The linear relationship given in equation (20) for determination of characteristic dimension is useful in notched strength evaluation of laminates. This study indicates that the present approach is superior to simply taking the average value of the characteristic length as in Ref. [1], determined from the experimental data and using this to predict the strength of other notch sizes.

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